Two-Dimensional Motion Formulas from Physics(Chapter 3)

The formula for displacement or position with constant acceleration "**a**", initial velocity $\mathbf{v}_0$, time "**t**", and initial position x_0 in one dimension is given below: $x = x_0 + v_0 t + \frac{1}{2}at^2$. This equation needs to be generalized for two-dimensional motion. In two dimensions, the displacement has a horizontal component and a vertical component as follows: $\underline{x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2}$ for the x-component and $\underline{y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2}$ for the y-component. In two dimensions, the velocity has a horizontal component and a vertical component as follows: $\underline{v_x = v_{x0} + a_x t}$ for the x-component and $\underline{v_y = v_{y0} + a_y t}$ for the y-component. v_{x0} = the initial velocity for the x-component; and v_{y0} = the initial velocity for the y-component. $\mathbf{a_x}$ = the constant acceleration for the x-component; and $\mathbf{a}_y$ = the constant acceleration for the y-component.

When dealing with projectile motion, a projectile is fired at an angle **θ** from the ground with initial velocity $\vec{v}$. The below figure shows the trigonometric relationship of the components of the initial velocity $\vec{v}$:

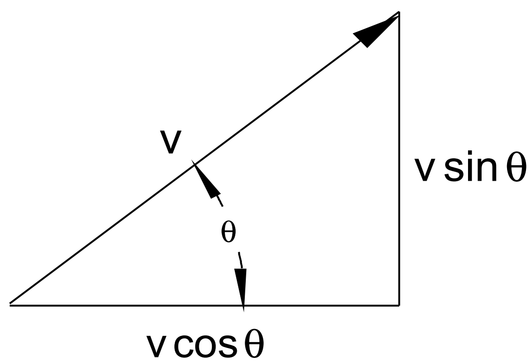

Figure 1: The initial velocity $\vec{v}$, a vector quantity, is pointing in the direction of the arrow. v = the magnitude of the velocity vector. The horizontal component = v_{x0} = v cos θ and the vertical component = v_{y0} = v sin θ.

The magnitude of the initial velocity vector $\vec{v}$ is v as shown above. In the horizontal direction, acceleration is assumed to be zero and the acceleration is assumed to be g =**9.8** m/s^2. Hence, $x = x_0 + v_{x0}t$, $v_x = v_{x0}$, $v_y = v_{y0} - gt$, and $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ are the equations that we will use to solve projectile motion. Usually, the starting point (x_0, y_0) is assumed to be (0, 0). Thus, $x = v_{x0}t$, $v_x = v_{x0}$, $v_y = v_{y0} - gt$, and $y = v_{y0}t - \frac{1}{2}gt^2$ are the equations we will use. Substituting $x = v_{x0}t$ into $y = v_{y0}t - \frac{1}{2}gt^2$ we have $y = \left[\frac{v_{y0}}{v_{x0}}\right]x - \frac{1}{2}g\left[\frac{v_{y0}x}{v_{x0}}\right]^2 \Rightarrow y = \left[\frac{v_{y0}}{v_{x0}}\right]x - \left[\frac{g}{2v_{x0}{}^2}\right]x^2$. In the case where

$v_{x0} = v\cos\theta$ and $v_{y0} = v\sin\theta$, $y = (\tan\theta)x - \left[\frac{g}{2v_0^2\cos^2\theta}\right]x^2$. Using the previous formula, (*set y = 0 and solve for x =* ***R****e*) we have the formula for the horizontal range **R**e on Earth is $R_e = \frac{v_0^2 \sin 2\theta_0}{g}$. The maximum height is $h_e = \frac{v_0^2 \sin^2\theta_0}{g}$.

Another kind of two-dimensional motion is circular motion of an object where the magnitude **v** of the velocity $\vec{v}$ of the object is constant. Just because the velocity $\vec{v}$ is pointing in one direction does not mean that the acceleration $\vec{a}$ is pointing in the same direction. The acceleration $\vec{a}$ is called the centripetal acceleration. The below figure shows the relationship between velocity $\vec{v}$ and centripetal acceleration $\vec{a}$ of an object in circular motion:

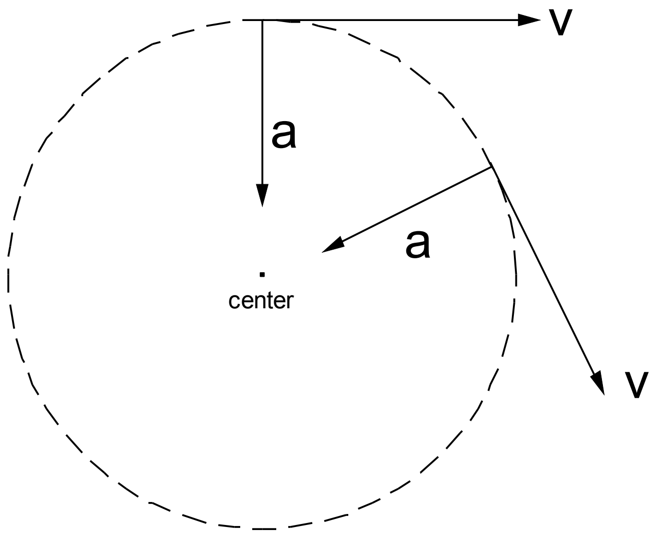

Figure 2: The above figure shows that the velocity and the centripetal acceleration vectors are perpendicular.

The magnitude of the centripetal acceleration $a = \frac{v^2}{r}$ where r = the radius of the circle and **v** = the magnitude **v** of the velocity $\vec{v}$ (speed).

Formulas that are useful in calculating the velocity's magnitude **v**(speed) is $F = \frac{1}{T}$ where F = the frequency in revolutions per second and T = the period = the amount of time to complete one revolution of the circle. Multiplying the frequency by 2π gives the angular speed $A = \frac{2\pi}{T}$ and multiplying A by the radius **r** gives the speed $v = \frac{2\pi r}{T}$.