SIMPSON'S RULE AND THE TRAPEZOIDAL RULE

<u>Two Rules for Approximating Integrals</u>:

1. $\int_a^b f(x)dx = \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)] = \text{Simpson's Rule}$

2. $\int_a^b f(x)dx = \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + f(x_n)] = \text{Trapezoidal Rule}$

Note that the function f(x) is defined on the interval [a,b]. The interval [a,b] is broken into n pieces of equal length. Thus, each piece has length $\frac{b-a}{n}$.

<u>Example</u>: Approximate $\int_1^2 (1/x)dx$ using Simpson's Rule and the Trapezoidal Rule where n = 4 (n = the number of pieces).

First split the interval [1,2] (a = 1 = lower limit of integration; b = 2 = upper limit of integration) into 4 pieces of equal length. Each piece has length (2–1)/4 = ¼.
Note x_0 = 1, x_0 = x_0 + ¼ = 5/4, x_2 = x_1 + ¼ = 3/2, x_3 = x_2 + ¼ = 7/4, x_4 = x_3 + ¼ = 2.
Thus, $f(x_0)$ = 1, $f(x_1)$ = 4/5, $f(x_2)$ = 2/3, $f(x_3)$ = 4/7, $f(x_4)$ = ½.

<u>By the trapezoidal rule</u>: $\int_1^2 (1/x)dx \approx$ 1/8(1 + 2(4/5) + 2(2/3) + 2(4/7) + ½) =

1/8(1 + 8/5 + 4/3 + 8/7 + ½) = (1/8 + 1/5 + 1/6 + 1/7 + 1/16) = .7

<u>By Simpson's rule</u>: $\int_1^2 (1/x)dx \approx$ 1/12(1 + 4(4/5) + 2(2/3) + 4(4/7) + ½) =

1/12(1 + 16/5 + 4/3 + 16/7 + ½) = (1/12 + 4/15 + 1/9 + 4/21 + 1/24) = .692

The true value is equal to $\int_1^2 (1/x)dx = \ln(x)\big|_1^2 = (\ln 2 - \ln 1) = \ln 2 = .693$