

FORMULAS AND PROCEDURES FOR ROTATIONS OF CURVES $f(x) = R$ ABOUT THE X-AXIS AND THE Y-AXIS.

1. The below formula is for rotating a curve $f(x) = R$ from **a** to **b** about the x-axis:

$$\pi \int_a^b R^2 dx = \pi \int_a^b f(x)^2 dx$$

Sometimes, the above formula is referred as the “disk method”.

If one wants to rotate about the y-axis from **c** to **d**, one has to solve for x in terms of y. Suppose that after doing this, $x = g(y) = R$. The above formula becomes

$$\pi \int_c^d R^2 dy = \pi \int_c^d g(y)^2 dy$$

2. The below formula is for rotating area between $f(x) = R =$ “**upper curve**” and $g(x) = S =$ “**lower curve**” from **a** to **b** about the x-axis:

$$\pi \int_a^b (R^2 - S^2) dx = \pi \int_a^b (f(x)^2 - g(x)^2) dx$$

Sometimes, the above formula is referred as the “washer method”. If one wants to rotate about the y-axis from **c** to **d**, one has to solve for x in terms of y for both curves. *Note that what may appear to be the upper curve looking at it from the x-axis may not be the upper curve looking at it from the y-axis.*

BE CAREFUL HERE! Suppose that after solving for x in terms of y for both curves, $x = h(y) = R =$ the upper curve and $x = w(y) = S =$ the lower curve. The above formula becomes

$$\pi \int_c^d (R^2 - S^2) dy = \pi \int_c^d (h(y)^2 - w(y)^2) dy$$

3. Sometimes, we want to rotate a piece of area about the y-axis, but we are unable to solve $y = f(x) =$ the upper curve = **R** and $y = g(x) =$ the lower curve = **S** for x in terms of y. We use the “cylindrical shells” method below:

$$2\pi \int_a^b x(R - S) dx = 2\pi \int_a^b x(f(x) - g(x)) dx$$

Think of this method as rotating about a circle. The circumference is $2\pi R$. Then just add an “x”.