One-Dimensional Motion Formulas from Physics(Chapter 2)

The formula for displacement or position with constant acceleration "**a**", initial velocity $\mathbf{v}_0$, time "**t**", and initial position x_0 in one dimension is given below: $x = x_0 + v_0 t + \frac{1}{2}at^2$. This equation is good for motion along the x-axis, any horizontal surface, or vertical(*up and down*) motion. From calculus, velocity is the derivative of position; i.e., $v = \frac{dx}{dt} = v_0 + at$.

When dealing with vertical motion, we use the acceleration of the Earth which is **9.8** m/s^2 where **m** = meters and **s** = time in seconds. We use the symbol "**–g**" instead of "**a**" in the above equation because objects eventually fall to the ground(*lose altitude*) when thrown upward. The above equations become $x = x_0 + v_0 t - \frac{1}{2}gt^2$ and $v = \frac{dx}{dt} = v_0 - gt$. The below example deals with vertical motion:

Example 1: A rock is thrown upward with an initial velocity of **10** m/s. Using the acceleration of gravity, **a**) How long will it take the rock to reach its maximum height? **b**) How high will the rock go up? **c**) How long will the rock take to come back down to the ground?

Solution: **a**) When the rock reaches its maximum height, the velocity of the rock $v = 0$; hence, $0 = v_0 - gt \Rightarrow v_0 = gt \Rightarrow 10 = 9.8t \Rightarrow t = \frac{10}{9.8} = \underline{\underline{1.02 \text{ sec.}}}$

b) Next, we can also assume that the initial position $x_0 = 0$. Since $x_0 = 0$ and the initial velocity $\mathbf{v}_0 = \mathbf{10}$ m/s, $x = 10t - \frac{1}{2}gt^2$. Because the rock reaches its maximum height in 1.02 sec, we can substitute **t** = 1.02 into the above equation and get the result below:

$t := 1.02 \quad x := 10 \cdot t - 4.9 \cdot t^2 \quad x = 5.102 \text{ meters}$

c) When the rock comes back to the ground, **x** will equal zero and **t** will not equal zero. Hence, $0 = 10t - 4.9t^2 = t(10 - 4.9t) \Rightarrow 4.9t = 10 \Rightarrow t = \underline{\underline{2.04 \text{ sec.}}}$

Since $v = v_0 + at$, $t = \frac{v - v_0}{a}$. Substituting that into $x = x_0 + v_0 t + \frac{1}{2}at^2$, we get

$$x = x_0 + v_0\left[\frac{v - v_0}{a}\right] + \frac{1}{2}a\left[\frac{v - v_0}{a}\right]^2 \Rightarrow (x - x_0) = \frac{vv_0 - v_0^2}{a} + \frac{v^2 - 2vv_0 + v_0^2}{2a}$$

$$\Rightarrow (x - x_0) = \frac{2vv_0 - 2v_0^2}{2a} + \frac{v^2 - 2vv_0 + v_0^2}{2a} \Rightarrow \underline{\underline{(x - x_0) = \frac{v^2 - v_0^2}{2a}}}$$

The above formula is used when time is not given, not needed, or when we want to establish a direct relationship between position and velocity. For vertical motion, substitute "**–g**" for "**a**". The above formula could have been used to solve part **b**) above. Notice that $x = \frac{-v_0^2}{-2g} = \frac{100}{19.6} = \underline{\underline{5.102 \text{ meters}}}.$

The next set of formulas is used when we want to figure out average velocity. One way to get the average velocity is to add the initial velocity $\mathbf{v_0}$ and the final velocity v and average them; i.e., $\bar{v} = \frac{v + v_0}{2}$. We can also get the average velocity by subtracting the initial position x_0 from x and dividing by "t"; i.e., $\bar{v} = \frac{x - x_0}{t}$.

Substituting, we have $\frac{v + v_0}{2} = \frac{x - x_0}{t}$. These formulas are useful when we are given a graph of time verses position. Solving for x we have $x = x_0 + \frac{(v + v_0)t}{2}$. The previous equation describes position as a function of velocity and time. The below example illustrates the use of the previous equation:

<u>Example 2</u>: A particle, moving on the x-axis starting at the origin with a speed of **5** m/s, increased its speed to **10** m/s in 20 seconds. How far from the origin did the particle go?

<u>Solution</u>: $x_0 = 0$ since the particle is starting at the origin. $v_0 = 5$ m/s and v = 10 m/s. Substituting, we have $x = \frac{(10 + 5)20}{2} = 150 \text{ meters}.$