Differences and Sums of Cubes

One topic that is studied in Algebra 2 courses in high school is **differences and sums of cubes**. In Algebra 1, you were taught how to factor **differences of squares**. This is shown below:

Notice that $a^2 - b^2 = (a - b)(a + b)$. An example is shown below:

Example 1(Difference of Squares): Factor $x^2 - 9$. Taking the square roots of x^2 and 9, we have x and 3. Writing (x 3) twice, we have (x 3)(x 3). Put a "+" sign in one of them and a "–" in the other, we have (x + 3)(x – 3).

Notice that there is no sum of squares; i.e., $a^2 + b^2$ is not factorable unless you have studied complex numbers.

For the difference of cubes, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. To illustrate this technique, we will outline the steps in the below example:

Example 2(Difference of Cubes): Factor $x^3 - 27$.

i) First, take the cube root of x^3 which equals x.
ii) Next, take the cube root of 27 which equals 3.
iii) Put the x and 3 together in parentheses and keep the "–" sign in $x^3 - 27$. The result is (x – 3).
iv) Square the x in (x – 3) to get x^2.
v) Multiply the x and the –3 in (x – 3) to get –3x and take the ***opposite sign*** to get 3x.
vi) Square the 3 in (x – 3) to get 9.
vii) Put the x^2, 3x and 9 together in parentheses to get (x^2 + 3x + 9).
viii) The final result is $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$.

For the sum of cubes, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$. Unlike the sum of squares, the **sum of cubes is factorable**. To illustrate this technique, we will outline the steps in the below example:

Example 3(Sum of Cubes): Factor $x^3 + 8$.

i) First, take the cube root of x^3 which equals x.
ii) Next, take the cube root of 8 which equals 2.
iii) Put the x and 2 together in parentheses and keep the "+" sign in $x^3 + 8$. The result is (x + 2).
iv) Square the x in (x + 2) to get x^2.
v) Multiply the x and the 2 in (x + 2) to get 2x and take the ***opposite sign*** to get –2x.
vi) Square the 2 in (x + 2) to get 4.
vii) Put the x^2, –2x and 4 together in parentheses to get (x^2 –2x + 4).
viii) The final result is $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$.