

DeMoivre's Theorem & Complex Numbers:

DeMoivre's theorem is used to raise complex numbers to various powers. All complex numbers can be written in the form $re^{i\theta}$ where r = magnitude of the complex number and $e^{i\theta} = \cos\theta + i\sin\theta$. θ = angle that the complex number makes in the complex plane.

To raise a complex number to a power "n", simply raise r to the n th power, and multiply the angle θ by "n" in the expression $e^{i\theta} = \cos\theta + i\sin\theta$ to get $r^n e^{in\theta} = r^n (\cos n\theta + i\sin n\theta)$. "n" can be any real number. For finding n th roots of numbers, note that $\cos\theta = \cos(\theta + 2k\pi)$ and $\sin\theta = \sin(\theta + 2k\pi)$ where k = any integer. Using this fact, we can write $e^{i\theta} = \cos\theta + i\sin\theta$ as $e^{i\theta} = \cos(\theta + 2k\pi) + i\sin(\theta + 2k\pi)$. To take the n th root of $re^{i\theta} = \cos\theta + i\sin\theta$, we take the n th root of r , and divide $(\theta + 2k\pi)$ by n in $e^{i\theta} = \cos(\theta + 2k\pi) + i\sin(\theta + 2k\pi)$ to get $r^{1/n} e^{i\theta/n} = r^{1/n} [\cos(\theta/n + 2k\pi/n) + i\sin(\theta/n + 2k\pi/n)]$. Run through the values $k = 1, 2, \dots, n$ to get the n n th roots of $re^{i\theta}$ = the complex number.

An example is done below: Find the 3 cube roots of $i = \sqrt{-1}$. First find the magnitude of i . The magnitude = $r = \sqrt{0^2 + 1^2} = 1$ because $a = 0$ and $b = 1$, all complex numbers are of the form $a + bi$, and the magnitude of $a + bi = \sqrt{a^2 + b^2}$. The cube root of $r = 1$ is 1. The angle θ that $i = \sqrt{-1}$ makes is 90° or $\pi/2$. Thus, $i = \cos(\pi/2 + 2k\pi) + i\sin(\pi/2 + 2k\pi)$. Dividing the angle by 3 we get $\sqrt[3]{i} = \cos(\pi/6 + 2k\pi/3) + i\sin(\pi/6 + 2k\pi/3)$. Running through $k = 1, 2, 3$ we get $\cos(5\pi/6) + i\sin(5\pi/6) = -\sqrt{3}/2 + i/2$ for $k = 1$; $\cos(3\pi/2) + i\sin(3\pi/2) = -i$ for $k = 2$; and $\cos(13\pi/6) + i\sin(13\pi/6) = \sqrt{3}/2 + i/2$ for $k = 3$.

DeMoivre's theorem is used to find the zeros of the polynomial $x^n - a = 0$ where "a" is a complex number. It can also be used to find some or all of the complex zeros of polynomials that are divisible by $x^n - a$.

DeMoivre's theorem has applications in electrical engineering and physics.