<u>Complex Numbers</u>:

From first grade math to the Algebra 2 course in high school, students work with real numbers. <u>Real numbers</u> are numbers that appear on the traditional number line; i.e., the positive and negative integers, the rational numbers, and the irrational numbers. These numbers are used in everyday applications. The traditional number line used in most math courses is illustrated below:

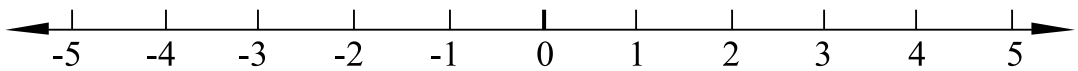

Figure 1: Here is the traditional number line used in most math courses.

When a student takes Algebra 1 in high school, he is told that taking square roots of negative numbers cannot be done. Take a look at **–9**. What is the square root of **–9**? The square root of **9** is **3**. Some people guess and say **–3**, but $(-3)^2 = 9$. In fact, raising a negative or positive number to the second power or any even power will result in a positive number. This problem pops up in the study of quadratic equations. Try to solve the quadratic equation below: $x^2 + x + 1 = 0$. Using the quadratic formula(a = **1**, b = **1**, and c = **1**), we have $x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$. Notice that we have a negative number in the radical. In most Algebra 1 courses, the teacher would say that the above equation has "no solution".

To handle this problem, mathematicians simply invented a number i defined as follows: $i^2 = -1$. (*Electrical engineers call this number j because i means current in their discipline*.) This leads to a larger class of numbers called the complex numbers. **<u>Complex</u> <u>numbers</u>** are defined as numbers of the form **a + bi** where **a** and **b** are real numbers. Notice that all real numbers are complex just by setting **b** = **0**. Also notice that **a =** the real part of the complex number, and **b =** the imaginary part of the complex number.

Notice that i follows a particular pattern for the exponents:
$i^1 = i,\ i^2 = -1,\ i^3 = i^2 i^1 = -1 \cdot i = -i, \text{and } i^4 = (i^2)^2 = (-1)^2 = 1$. The pattern repeats itself for exponents of 5 or higher. Notice that $i^5 = i,\ i^6 = -1,\ i^7 = -i, \text{and } i^8 = 1$. In general, if one wants to know what i^n equals where **n** > 4, just divide **n** by 4 and <u>keep the remainder</u>. An example is give below: $i^{123} = i^3 = -i$ because dividing 4 into 123 produces a quotient of 30 with a remainder of 3; i.e., $123 \bmod 4 = 3$.

Complex numbers can be added or subtracted just like regular algebra expressions where i is treated like a variable; i.e., (**a + b**i) ± (**c + d**i) = (**a** + **c**) ± (**b** + **d**)i. Two examples are below: $(3 + 4i) + (1 + i) = 4 + 5i$ and $(7 + 2i) - (3 + i) = 4 + i$.

Complex numbers can be multiplied just like regular algebra expressions where i is treated like a variable; i.e., the "foil" method can be used to multiply two complex numbers. An example is given below: $(3 + 4i)(1 + i) = 3 + 3i + 4i + 4i^2 = 3 + 3i + 4i - 4 = \underline{\underline{-1 + 7i}}$. Because of the pattern of the exponents of i and because all complex numbers are of the form **a + b**i, **<u>never leave exponents > 1 on i</u>**. Teachers and professors will mark a problem wrong or incomplete if any term of a complex number or expression has an exponent > **1** on i.